IPPP/10/83; DCPT/10/166

Lower Bounds on the Smallest Lepton Mixing Angle

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We give minimal values for the smallest lepton mixing parameter $|U_{e3}|$, applying 2-loop renormalization group equations in an effective theory approach. This is relevant in scenarios that predict an inverted neutrino mass spectrum with the smallest mass and $|U_{e3}|$ being zero at tree level, a situation known to be preserved at 1-loop order. At 2-loop, $|U_{e3}|$ is generated at a level of 10^{-12}–10^{-14}. Such small values are of interest in supernova physics. Corresponding limits for the normal mass ordering are several orders of magnitude larger. Our results show that $|U_{e3}|$ can in general expected be to be non-zero.

PACS numbers: 14.60.Pq; 11.10.Gh

The subject of lepton mixing and neutrino mass has entered the precision era. It is confirmed that there are three flavors of neutrinos which mix among themselves because the flavor states are not identical to the states with distinct masses m_1, m_2 and m_3. The parameters describing the mixing are the three angles θ_{12}, θ_{23}, θ_{13} and a phase δ. Two more phases $\varphi_{1,2}$ are needed if neutrinos are Majorana particles, which is the case in basically all extensions of the Standard Model, and also from an effective field theory point of view. The two mixing angles θ_{12} and θ_{23} as well as the mass squared difference $\Delta m^2_\odot \equiv m_2^2 - m_1^2$ and the magnitude of $\Delta m^2_{\rm A} \equiv m_3^2 - m_2^2$ have been determined with high accuracy, and further improvement is foreseen with a variety of experimental approaches [1]. However, of particular interest is the parameter $|U_{e3}| = \sin\theta_{13}$, which describes the electron neutrino content in the heaviest (lightest) neutrino mass-eigenstate, if the normal (inverted) mass ordering is realized, *i.e.* if $\Delta m^2_{\rm A} > 0$ ($\Delta m^2_{\rm A} < 0$). The main feature of $|U_{e3}|$ is its smallness compared to the other mixing angles which makes it difficult to be measured experimentally [2]. Three independent groups performing global fits of the world's neutrino data find 3σ limits of $|U_{e3}|^2 \leq 0.046$ [3], ≤ 0.053 [4], and ≤ 0.043 (0.047) [5], where the two values for Ref. [5] stem from two slightly different analyses of solar data. $|U_{e3}|$ is of fundamental importance as it is crucial to rule out a large number of models constructed to explain the peculiar structure of lepton mixing [6], CP violation in oscillations depends crucially on $|U_{e3}|$, and it also plays an important role in neutrino-less double beta decay [7]. Although some rather weak hints on a non-zero value exist [3] at $\sim$1.5σ, all existing data are well compatible with $|U_{e3}| = 0$.

The smallness of $|U_{e3}|$ is most naturally explained by some flavor symmetry [8–10] predicting $|U_{e3}|$ to vanish at some high scale Λ. In general, the symmetry is broken and potentially generates small but non-zero values of $|U_{e3}|$ at the low scale λ relevant for the oscillation experiments. Radiative corrections are one inevitable source of this breaking. In this letter we discuss the minimally induced values of $|U_{e3}|$ through renormalization group (RG) evolution, in order to investigate whether non-zero values of $|U_{e3}|$ can be expected in general. Given the huge amount of experimental activity and the theoretical importance of this parameter, it is obviously of interest to investigate whether non-zero values can be expected on general grounds. We therefore especially focus on the case of an inverted mass ordering with $m_3 = |U_{e3}| = 0$, which is a fixed point at 1-loop. It is shown that at 2-loop order this is no longer a fixed point, and we argue that it leads to the lowest possible value of $|U_{e3}|$, if extreme fine-tuning of the parameters is neglected. The order of magnitude of $|U_{e3}|$ at low scale lies between 10^{-12} and 10^{-14}. This is, of course, beyond the reach of the planned neutrino oscillation facilities, but of interest in supernova physics. It is furthermore easy to rule out this possibility. We also give the corresponding bounds on $|U_{e3}|$ for the normal mass-ordering, which happens to be several orders of magnitude larger.

The 1-loop RG evolution of $\theta_{13} = \sin^{-1}|U_{e3}|$, up to first order, is given by [11, 12]

$$\begin{aligned}\dot\theta_{13} = \frac{C\,y_\tau^2}{32\pi^2}\,\sin 2\theta_{12}\,\sin 2\theta_{23}\,\frac{m_3}{\Delta m^2_{\rm A}\,(1+R)} \times \\ \left[m_1\cos(\varphi_1-\delta) - (1+R)m_2\cos(\varphi_2-\delta) - R m_3\cos\delta\right]\,,\end{aligned} \tag{1}$$

where the dot denotes $\mathrm{d}/\mathrm{d}t = \mu\,\mathrm{d}/\mathrm{d}\mu$ with $t \equiv \ln(\mu/\Lambda)$, μ being the renormalization scale. One has $C = -\frac{3}{2}$ ($C = 1$) in the Standard Model (SM) (Minimal Supersymmetric SM (MSSM)), $R = \Delta m^2_\odot/\Delta m^2_{\rm A}$, and the tau Yukawa coupling is $y_\tau = \sqrt{2}\,(m_\tau/\mathrm{GeV})/246 \simeq 0.010$ in the SM and $0.010\,(1+\tan^2\beta)$ in the MSSM. The electron and muon Yukawa couplings $y_{e,\mu}$ have been neglected here. Eq. (1) shows that the evolution of $|U_{e3}|$ is proportional to m_3. Moreover at 1-loop, one has $\dot m_i \propto m_i\,(i = 1,2,3)$ and thus $|U_{e3}|(\mu) = m_3(\mu) = 0$ is a fixed point of RG evolution: these quantities remain zero throughout the RG evolution. The stability of this scenario under radiative corrections remains true even if $y_{e,\mu}$ are taken into account [9].

To understand the existence of the fixed point in a more general form, we consider the neutrino mass matrix m_ν for $m_3 = |U_{e3}| = 0$, which obeys a structure denoted as "scaling" [10]:

$$m_\nu^{\rm scaling} = \begin{pmatrix} a & b & b/c \\ \cdot & d & d/c \\ \cdot & \cdot & d/c^2 \end{pmatrix} \,, \qquad (2)$$

where $\tan^2\theta_{23} = |1/c^2|$ and a, b, d are determined by $m_{1,2}$, θ_{12}, and the Majorana phase difference $\Delta\varphi = \varphi_2 - \varphi_1$. At 1-loop, the RG equation of m_ν is given as [11, 12]

$$\dot{m}_\nu = \frac{1}{16\pi^2}\left[\alpha_\nu\, m_\nu + C\left(Y^T\, m_\nu + m_\nu\, Y\right)\right] , \qquad (3)$$

where $Y = Y_\ell\, Y_\ell^\dagger$ with $Y_\ell = {\rm diag}(y_e, y_\mu, y_\tau)$ being the charged lepton Yukawa matrix. The parameter α_ν is a function of gauge and Yukawa couplings (as well as the Higgs self-couplings in the SM), which does not lead to modifications of the mixing matrix elements. From Eq. (3) RG evolution of an element $(m_\nu)_{\alpha\beta}$ becomes [13]

$$(\dot{m_\nu})_{\alpha\beta} = \frac{1}{16\pi^2}\left[\alpha_\nu + C\left(y_\alpha^2 + y_\beta^2\right)\right](m_\nu)_{\alpha\beta} \,, \qquad (4)$$

which at low scale leads to the RG-modified mass matrix

$$m_\nu = I_{\alpha_\nu}\begin{pmatrix} (m_\nu^0)_{ee}\, I_e^2 & (m_\nu^0)_{e\mu}\, I_e\, I_\mu & (m_\nu^0)_{e\tau}\, I_e\, I_\tau \\ \cdot & (m_\nu^0)_{\mu\mu}\, I_\mu^2 & (m_\nu^0)_{\mu\tau}\, I_\mu\, I_\tau \\ \cdot & \cdot & (m_\nu^0)_{\tau\tau}\, I_\tau^2 \end{pmatrix} , \qquad (5)$$

where m_ν^0 denotes the mass-matrix at the high-scale and

$$I_\alpha = \exp\left(\tfrac{C}{16\pi^2}\int_\Lambda^\lambda y_\alpha^2\, {\rm d}t\right) , \qquad (6)$$

with $\alpha, \beta \in \{e, \mu, \tau\}$. Eq. (5) shows that the scaling property of $m_\nu^{\rm scaling}$ is preserved by the RG evolution since the second and third column remain proportional to each other. The atmospheric neutrino mixing changes to $\tan^2\theta_{23} = I_\tau^2/(I_\mu^2\,|c^2|)$. Thus $m_3 = |U_{e3}| = 0$ remains valid at all energies during 1-loop running. Therefore, any non-zero $|U_{e3}|$ generated at 2-loop can be considered to be the smallest guaranteed value for $|U_{e3}|$, which obviously is of interest in its own right, and should in the end represent the final precision goal for any search. If this 2-loop value cannot vanish, it follows that in general $|U_{e3}|$ can be expected to be non-zero.

The largest contribution in Eq. (6) will come from $|C|\, y_\tau^2/(16\pi^2) \simeq 9.5 \times 10^{-7}$ for the SM and $\simeq 6.3 \times 10^{-7}\,(1 + \tan^2\beta)$ for the MSSM. I_α in Eq. (6) can thus be approximated as $I_\alpha \simeq 1 + \frac{C}{16\pi^2}\, y_\alpha^2\, \ln\frac{\lambda}{\Lambda} \equiv 1 + \epsilon_\alpha$.

At 2-loop, the relevant term in the RG evolution equation of m_ν in case of the SM is [14]

$$\dot{m}_\nu = \frac{2}{(16\pi^2)^2}\, Y^T\, m_\nu\, Y \,, \qquad (7)$$

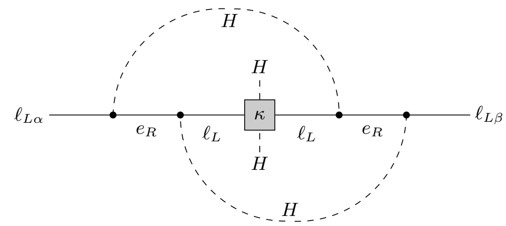

FIG. 1: Rank-changing 2-loop diagram in the SM which breaks the 1-loop RG invariance $m_3 = |U_{e3}| = 0$. ℓ_L (e_R) are left-(right-)handed leptons and H is the Higgs doublet.

which arises from the diagram shown in Fig. 1. In the MSSM, there are no such terms (because of the non-renormalization theorem) unless supersymmetry (SUSY) is broken [15]. Hence in the remaining part of this paper we will consider the SM, unless otherwise specified. From Eq. (7) the evolution of $(m_\nu)_{\alpha\beta}$ can be written as

$$(\dot{m_\nu})_{\alpha\beta} = \frac{2}{(16\pi^2)^2}\, y_\alpha^2\, y_\beta^2\, (m_\nu)_{\alpha\beta} \,, \qquad (8)$$

and the mass-matrix m_ν gets modified to

$$m_\nu \propto \begin{pmatrix} (m_\nu^0)_{ee}\, I_e^2\, I_{ee} & (m_\nu^0)_{e\mu}\, I_e\, I_\mu\, I_{e\mu} & (m_\nu^0)_{e\tau}\, I_e\, I_\tau\, I_{e\tau} \\ \cdot & (m_\nu^0)_{\mu\mu}\, I_\mu^2\, I_{\mu\mu} & (m_\nu^0)_{\mu\tau}\, I_\mu\, I_\tau\, I_{\mu\tau} \\ \cdot & \cdot & (m_\nu^0)_{\tau\tau}\, I_\tau^2\, I_{\tau\tau} \end{pmatrix} . \qquad (9)$$

The new parameters $I_{\alpha\beta}$ are given by

$$\begin{aligned} I_{\alpha\beta} &= \exp\left\{\tfrac{2}{(16\pi^2)^2}\int y_\alpha^2\, y_\beta^2\, {\rm d}t\right\} \\ &\simeq 1 + \tfrac{2}{(16\pi^2)^2}\, y_\alpha^2\, y_\beta^2\, \ln\tfrac{\lambda}{\Lambda} \equiv 1 + \epsilon_{\alpha\beta} \,. \end{aligned} \qquad (10)$$

As can be seen, 2-loop running increases the rank of m_ν from 2 to 3. In Eq. (10) the largest contribution comes from $|\epsilon_{\tau\tau}| \simeq 2.0 \times 10^{-11}$, and thus numerically is of the same order as that of ϵ_e at 1-loop. Here we have taken $\lambda = 10^2$ GeV and $\Lambda = 10^{12}$ GeV, which will be used throughout this paper. Obviously, the scale of $\epsilon_{\tau\tau}$ sets the value for $|U_{e3}|$ generated at 2-loop order.

To explicitly calculate the implied smallest value of $|U_{e3}|$ after 2-loop running, we consider the scaling scenario $m_3 = |U_{e3}| = 0$ at the high scale Λ. It suffices to take only $\epsilon_{\tau\tau}$ into account and neglect all other $\epsilon_{\alpha\beta}$ in Eq. (10). As a result of the change in rank of m_ν, the 2-loop running, unlike the 1-loop case, gives rise to a non-zero smallest neutrino mass

$$\begin{aligned} m_3 &= \tfrac{1}{4}\,\sin^2 2\theta_{23}\left(m_2\,\cos^2\theta_{12}\, e^{i\Delta\varphi} + \sin^2\theta_{12}\, m_1\right)|\epsilon_{\tau\tau}| \\ &\simeq \tfrac{1}{4}\, m_2\,\sin^2 2\theta_{23}\left(\cos^2\theta_{12}\, e^{i\Delta\varphi} + (1 + \tfrac{R}{2})\,\sin^2\theta_{12}\right)|\epsilon_{\tau\tau}| \,. \end{aligned}$$

The order of m_3 is $\simeq m_2\,|\epsilon_{\tau\tau}|/4 \sim 2.5 \times 10^{-13}$ eV for $m_2 \simeq 0.05$ eV. It confirms the result from Ref. [14].

The 2-loop evolution equation for $|U_{e3}|$ in the scaling scenario can be obtained from Eq. (7), following the pro-

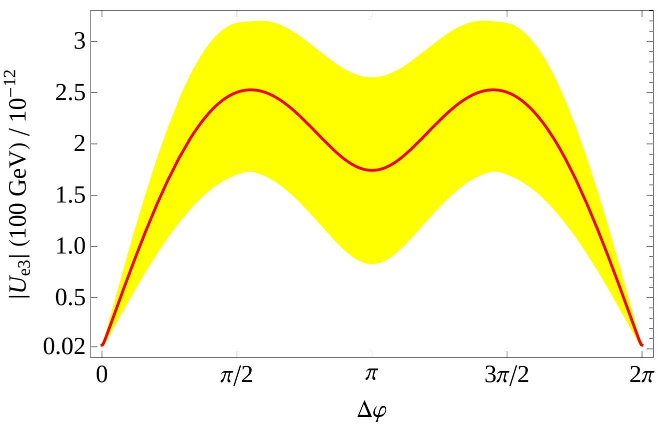

FIG. 2: Generated value of $|U_{e3}|$ at $\lambda = 100\,\mathrm{GeV}$ assuming a vanishing value at $\Lambda = 10^{12}\,\mathrm{GeV}$ in the leading-log approximation. The (solid) red line corresponds to fixing all mixing parameters to the best-fit values and the yellow region shows the 3σ region, which are taken from [4].

cedure of [16] (see also [17]), to be

$$\frac{\mathrm{d}}{\mathrm{d}t}|U_{e3}| \simeq |\dot{U}_{e3}| \simeq \frac{y_\tau^4}{2\,(16\pi^2)^2}\frac{s_{23}^2}{m_1\,m_2}\sin 2\theta_{12}\,\sin 2\theta_{23}\times \left|m_1 - m_2\,e^{i\Delta\varphi}\right|\left|m_1\,s_{12}^2 + m_2\,c_{12}^2\,e^{i\Delta\varphi}\right|, \tag{11}$$

in the limit $y_{e,\mu} \ll y_\tau$. The β-function of $|U_{e3}|$ does not depend on the Dirac CP phase δ for vanishing $|U_{e3}|$ [11], which has been discussed in detail in [18]. From Eq. (11), the typical order of magnitude of $|U_{e3}|$ at low scale λ is $|U_{e3}| \simeq y_\tau^4/(4\,(16\pi^2)^2)\,\ln\Lambda/\lambda \simeq 10^{-12}$. Fig. 2 shows the dependence of $|U_{e3}|$ on the Majorana phase difference $\Delta\varphi$. The maximal value of $|U_{e3}|$ is obtained for $\cos\Delta\varphi = (\tan^2\theta_{12} - 1)(m_2^2 - m_1^2\,\tan^2\theta_{12})/(4\,m_1\,m_2\,\tan^2\theta_{12})$, and gives

$$|U_{e3}| \simeq 3.2\times 10^{-12}\,, \tag{12}$$

while the minimum value, obtained for $\Delta\varphi = 0$, is

$$|U_{e3}| \simeq 2.0\times 10^{-14}\,. \tag{13}$$

It is suppressed with respect to Eq. (12) by a factor of order $R \simeq 1/30$, which remains true even if $y_{e,\mu}$ are taken into account. It is clear that a 3-loop effect could not cancel this suppressed value, since it would be further suppressed by $1/(16\pi^2) \ll R$.

One possibility for a smaller value of $|U_{e3}|$ would be if an initially non-zero value accidentally runs to (almost) zero in the course of its evolution which, of course, will need an extreme amount of fine-tuning. Another possibility arises when the high energy completion of the effective theory is considered and the RG effects in the complete theory conspire with the effective running to give tiny values, *e.g.* threshold effects in case of seesaw models. However, that will depend on the particular high energy extension considered, and will again involve extreme fine-tuning. For example, in seesaw models threshold effects generally lead to larger RG running than the running in the effective theory [17], and hence a fine-tuned cancellation will be required to produce the tiny $|U_{e3}|$ of relevance in this discussion. Hence it is natural to claim that the 2-loop generated $|U_{e3}|$ as given in Eq. (11) is the smallest value one should expect. Moreover, it shows that non-zero $|U_{e3}|$ can be expected on general grounds.

As previously mentioned, $|U_{e3}|$ is not generated at 2-loop in the MSSM, as long as SUSY is unbroken. However, analogous to the discussion for m_3 in [14], $|U_{e3}|$ is generated as SUSY is broken and receives contributions of two different origins. First, there is a 2-loop contribution with sleptons and selectrons in the loop as shown in Fig. 1 of [14]. Although it is not enhanced by a large logarithm, it depends on $(1+\tan^2\beta)^2$ along with a complicated order one function of sparticle masses. Second, there can be corrections due to the the SUSY breaking operator $\tilde{L}H_u\tilde{L}H_u$ [19], which leads to $|U_{e3}|$ of similar order as the 2-loop contribution. Thus while a general prediction is not possible, it suffices to estimate the order of magnitude to be

$$|U_{e3}|^{\mathrm{MSSM}} \sim |U_{e3}|^{\mathrm{SM}}\,(1+\tan^2\beta)^2\,/\ln\frac{\Lambda}{\lambda}\,, \tag{14}$$

where $|U_{e3}|^{\mathrm{SM}}$ corresponds to the values in the SM discussed above. Thus even for moderate $\tan\beta$, $|U_{e3}|^{\mathrm{MSSM}}$ will be larger than the corresponding SM value.

Can such values be tested? With the present view, the ultimate experiment in order to study neutrino oscillations will be a neutrino factory. However, the facilities currently under study [1] have a 3σ discovery potential on $|U_{e3}|$ of at most 1.5×10^{-3}, and hence the 2-loop generated tiny $|U_{e3}|$ is far beyond its scope.

However, tiny non-zero values of $|U_{e3}|$, as discussed here, may have observable consequences in the neutrino spectra emitted by supernovae. Extremely high neutrino densities around the neutrinosphere give rise to so-called collective effects [23], of which "spectral swapping" is the one where ν_e and $\nu_{\mu,\tau}$ swap their energy spectra at high (low) energies but keep their spectra at low (high) energies in case of inverted (normal) hierarchy. Here "high" and "low" energies are understood relative to a critical energy E_C, which decreases with $|U_{e3}|$ in the normal ordering, while is independent of $|U_{e3}|$ if the mass-ordering is inverted [24]. The effect can be well understood with the analogy of an inverted pendulum [25], which turns around by the slightest instability, generated by a non-zero $|U_{e3}|$ in this case. In case of inverted mass-ordering, the spectral swap occurs regardless of how small but non-zero $|U_{e3}|$ is, the impact of $|U_{e3}|$ being only to introduce a logarithmic dependence on the radius at which the conversion sets in. Ref. [24] discusses in this context values

of $|U_{e3}|$ down to 10^{-70}, 56 orders of magnitude smaller than the lower limit obtained here.

While the discussion here may be a bit too simplified (the effects may depend on supernova details, they can be also induced by other effects like an asymmetry in the primary ν_μ–ν_τ fluxes as well as radiatively induced matter effects for $\nu_{\mu,\tau}$ [26], etc.), it catches the main point to be made here: tiny values of $|U_{e3}|$, in particular the ones generated by 2-loop running in the inverted hierarchy, are expected to have observable consequences in supernova physics. The lower bounds on $|U_{e3}|$ presented here may be helpful in analyses of supernova neutrino analyses.

The scenario leading to the smallest possible value of $|U_{e3}|$ can easily be tested or ruled out, not only by long-baseline oscillation experiments pinning down the mass ordering, but also via neutrino mass related observables: for $m_3 = |U_{e3}| = 0$ the effective mass governing neutrinoless double beta decay is simply

$$\langle m \rangle = \sqrt{|\Delta m^2_{\rm A}|}\,\sqrt{1 - \sin^2 2\theta_{12}\,\sin^2 \Delta\varphi/2}\,. \tag{15}$$

The Majorana phase difference $\Delta\varphi$ is the same as the one determining the magnitude of $|U_{e3}|$ and is, in principle, measurable by precise measurements of double beta decay [20]. In contrast to its value in the normal ordering, $\langle m \rangle$ is bounded from below by $\sqrt{|\Delta m^2_{\rm A}|}\,\cos 2\theta_{12}$, which is testable in future experiments [21]. In what regards cosmology, the sum of neutrino masses is about $2\sqrt{|\Delta m^2_{\rm A}|} \simeq 0.1$ eV, which is twice as high as its value in the normal ordering, and might be testable in future measurements as well [22].

In case of normal ordering the minimal $|U_{e3}|$ can trivially be obtained from the 1-loop running given in Eq. (1). The minimal value is obtained with $\varphi_1 = \varphi_2 = \pi$ to be

$$|U_{e3}| = \frac{|C|\,y_\tau^2}{32\pi^2}\sin 2\theta_{12}\,\sin 2\theta_{23}\,\frac{m_3\,(m_2 - m_1)}{(m_1 + m_3)\,(m_2 + m_3)}\,\ln\frac{\Lambda}{\lambda}\,, \tag{16}$$

which quantifies to $|U_{e3}| \sim 1.1 \times 10^{-6}$ in the SM and $\sim 1.6 \times 10^{-6}\,(1 + \tan^2\beta)$ in the MSSM. Interestingly, $|U_{e3}|$ is smallest for quasi-degenerate neutrinos and not for $m_1 = 0$. Introducing the common mass scale m_0 of the quasi-degenerate neutrinos, the lowest values are decreased by a factor of

$$\sim \frac{8 m_0^2}{\left(\sqrt{\Delta m^2_\odot} + \sqrt{\Delta m^2_{\rm A}}\right)\sqrt{\Delta m^2_\odot}} \simeq 1600\left(\frac{m_0}{0.3\,{\rm eV}}\right)^2$$

and hence reduce the scale of $|U_{e3}|$ by at most three orders of magnitude, and still stay three orders above the 2-loop value in the inverted hierarchy.

In summary, we have analyzed the 2-loop running of U_{e3} in the effective low energy theory and showed that it gives the minimal value of $|U_{e3}|$ in scenarios of inverted mass hierarchy with $U_{e3} = 0$ at high scale. We focussed on this scenario because at 1-loop there is no generation of a non-zero value, and because it is of interest to investigate whether non-zero $|U_{e3}|$ can be expected in general. The order of magnitude of $|U_{e3}|$ generated in this way is between 10^{-12} and 10^{-14} depending on the Majorana phases. Though such small values are beyond the reach of currently foreseen neutrino oscillation experiments, they are obviously of fundamental interest, and possess applications in supernova physics.

Acknowledgments: The work of W.R. was supported by the ERC under the Starting Grant MANITOP and by the DFG in the project RO 2516/4-1 as well as in the Transregio 27. S.R. would like to thank Prof. Yuval Grossman for support and hospitality. M.A.S. expresses his gratitude to Manfred Lindner for hospitality at the MPIK.

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